St James' RC Primary School

Guidance on calculating

Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The Characteristics of a Primary Numerate child are:

A child who…

- Has a sense of the size of a number
- Knows where numbers fit in the number system
- Recalls number facts and uses these to aid mental calculations
- Uses a range of methods for calculating
- Uses a calculator when needed
- Makes sense of number problems and adopts strategies to solve them
- Makes estimates and checks answers
- Discusses strategies used
- Explains their mathematical ideas
- *Feels confident*

Aim

The overall aim, in teaching calculation at St James, is that by the end of key Stage 2, all children should be able to use an efficient written method for each operation with confidence and understanding.

Year by year Objectives for Calculating

Year Group	Objectives
Foundation Stage (Reception class)	• Say which number is one more or one less than a given number. • Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. • solve problems, including doubling, halving and sharing
Year 1	• read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs • represent and use number bonds and related subtraction facts within 20 • add and subtract one-digit and two-digit numbers to 20, including zero • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 =? – 9. • solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Year 2	• solve problems with addition and subtraction: • use concrete objects and pictorial representations, including those involving numbers, quantities and measures • apply their increasing knowledge of mental and written methods • recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers, adding three one-digit numbers • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. • recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • solve problems involving multiplication and division, using materials, arrays, repeated addition, mental
Year 3	• add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds, add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction. • recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Year 4	• add and subtract numbers with up to 4 digits using the formal written methods of

	columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. • recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.
Year 5	• add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. • identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19 • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
Year 6	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • perform mental calculations, including with mixed operations and large numbers • identify common factors, common multiples and prime numbers • use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Prior Learning

Consistency of approach is crucial to children's understanding of written calculations.
If children are introduced to compact methods before their understanding is developed they will not be successful.

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient **counting strategies** and a secure knowledge of **place value** and **number facts**. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides **practice** and **consolidation** of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

Operation	Prior Learning – children need to be able to:
Addition	• recall all addition pairs to 9 + 9 and complements in 10; • add mentally a series of one-digit numbers, such as 5 + 8 + 4; • add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of **place value**; • partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways. • understand that addition can be done in any order.
Subtraction	• recall all addition and subtraction facts to 20; • subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of **place value**; • partition two-digit and three-digit numbers into multiples of one hundred, ten and ones in different ways (e.g. partition 74 into 70 + 4 or 60 + 14). • Know the next 10, 100 and 1000 for any number • Know the previous 10, 100 and 1000 for any number
Multiplication	• recall all multiplication facts to 12 × 12; • partition number into multiples of one hundred, ten and ones; • work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of add two or more single-digit numbers mentally; • add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value; • add combinations of whole numbers using the column method (see above).
Division	• understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient; • partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways; • recall multiplication and division facts to 12 x 12, recognise multiples of one-digit numbers and divide

	multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value; • know there may be and how to find a remainder – for example, find the remainder when 48 is divided by 5; • understand and use multiplication and division as inverse operations (fact families) • understand division as repeated subtraction; • subtract numbers using the column method.

Learning key facts

Year	Key facts
EYFS (Reception)	Addition and subtraction facts to 10 Doubles facts to 10
Year 1	Addition and subtraction facts to 20 Addition and subtraction of multiples of 10 to 100
Year 2	Ten times table Five times table Two times table
Year 3	Four times table Eight times table Three times table
Year 4	Six times table Nine times table Seven times table

Progression towards a formal written method:

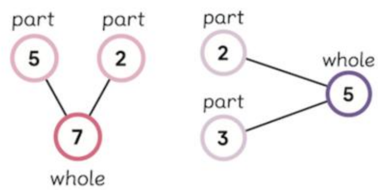
- Establish understanding of the concept through thorough practical work with a range of manipulatives.
- Establish mental methods, based on a good understanding of place value in numbers.
- Solve calculations in a horizontal format.
- Children set out written calculations, using expanded layouts that record their mental methods, initially alongside manipulatives and then stand alone.
- As children become more confident, they refine the written record into a more compact and efficient method.
- Extend to larger numbers and to decimals. (Children may initially need to return to a more expanded layout.)

Addition

All children are introduced to addition through practical and visual means. When children are ready to start recording their work they will progress as follows:

Stage 1

Part-part-whole model. To introduce the idea that two numbers can make another number



Stage 2

Bar model – to introduce understanding and reasoning about addition and what the equals sign means.

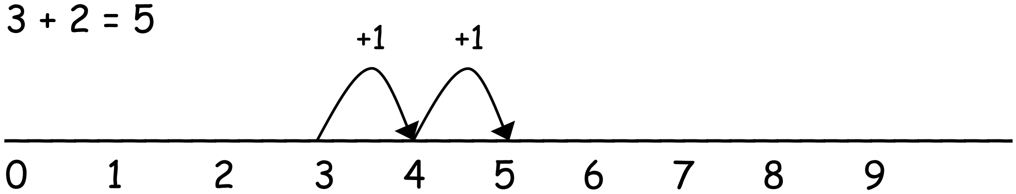
10	
5	5

5 + 5 = 10
10 = 5 + 5

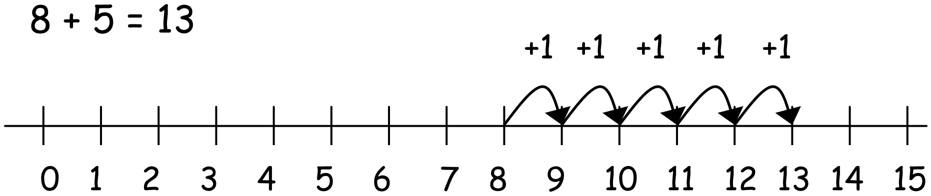
Stage 3

Children use numbered lines to support their calculating by counting on in ones.

3 + 2 = 5



8 + 5 = 13



Stage 4

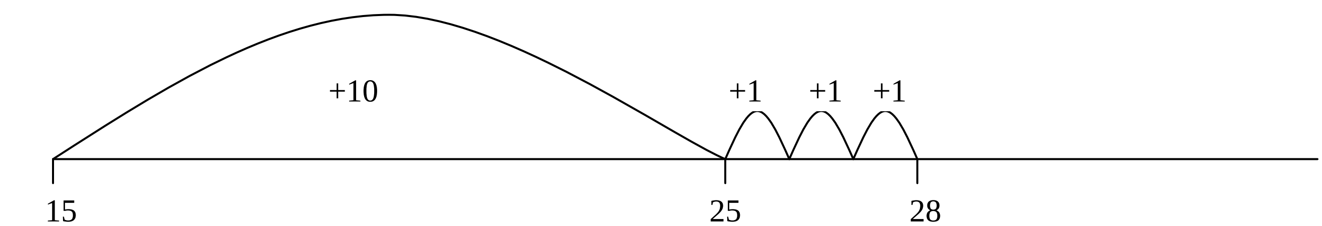
Children use empty number line to record steps in their calculating.

Including <u>bridging to ten</u>:

8 + 7 = 15

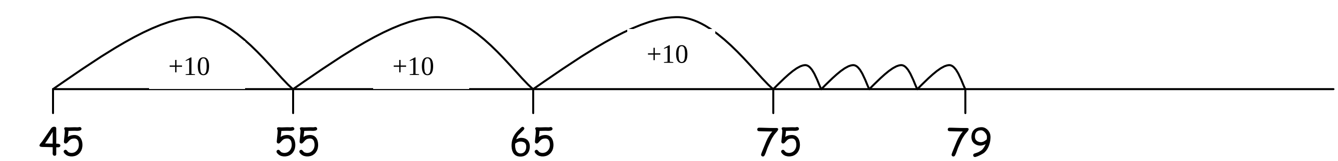
<u>Partitioning</u>:

13+15=



<u>Adding larger numbers</u>:

45+34=



Stage 5

Add the ones and then the tens to form partial sums and then add these partial sums. Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods

45 + 28 = 73

40 5 20 8

40 + 20 = 60 5 + 8 = 13

60 + 13 = 73

Becoming:

45 + 28 = 73

60 13

73

Stage 6

Expanded method in columns showing the addition of the tens to the tens and the ones to the ones separately.

34+23=

30	4
20	3
50	7

Leading to:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array} \qquad \begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$

$$\begin{array}{rrl} & 378 & \\ + & 475 & \\ \hline & 700 & (300 + 400) \\ & 140 & (70 + 70) \\ & 13 & (8 + 5) \\ \hline & 853 & \end{array} \qquad \begin{array}{rrl} & 9.72 & \\ + & 6.59 & \\ \hline & 15.00 & (9 + 6) \\ & 1.20 & (0.7 + 0.5) \\ & 0.11 & (0.02 + 0.09) \\ \hline & 16.31 & \end{array}$$

STAGE 7:

Decomposition (the column method)

$$\begin{array}{rr} & {\scriptstyle 1\,1} \\ & 47 \\ + & 76 \\ \hline & 123 \\ \hline \end{array} \qquad \begin{array}{rr} & {\scriptstyle 1\,1} \\ & 258 \\ + & 87 \\ \hline & 345 \\ \hline \end{array} \qquad \begin{array}{rr} & {\scriptstyle 1\,1} \\ & 366 \\ + & 458 \\ \hline & 824 \\ \hline \end{array}$$

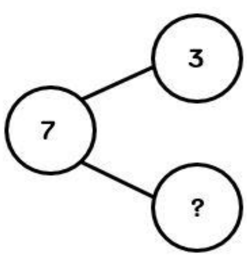
Recording is further reduced and remains efficient when used with larger whole numbers and decimal numbers.

Subtraction

Following practical and visual methods:

Stage 1

Part-part-whole model – to understand you can take one part and the other part will be left.



Stage 2

Bar model - to understand and reason what subtraction means in relation to the equals sign.

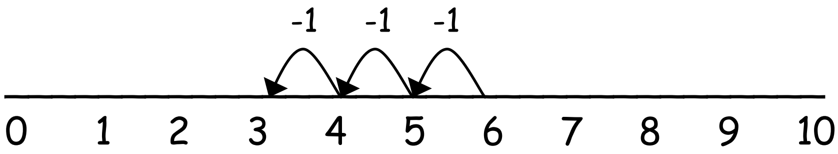
<table>
<tr><td colspan="2">6</td></tr>
<tr><td>3</td><td>?</td></tr>
</table>

6 – 3 = 3

Stage 3

Children use numbered lines to support their calculating by counting back in ones.

6 – 3 = 3



Stage 4

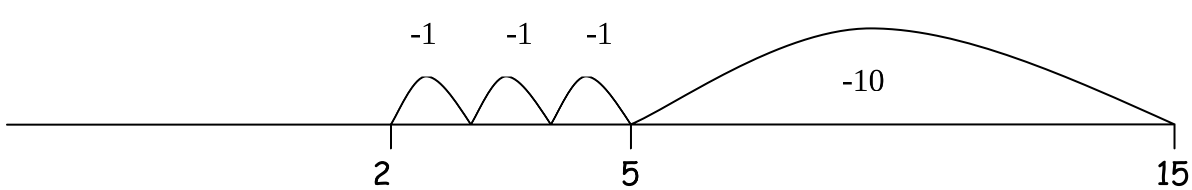
Children use empty number line to record steps in their calculating.

Bridging:

15 – 7 = 8

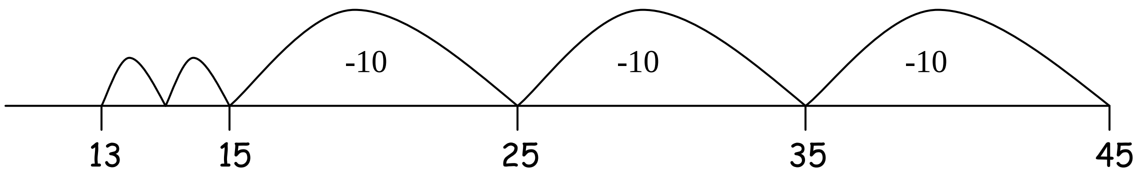
Partitioning:

15-13=



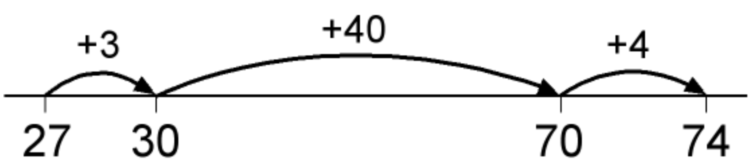
Subtracting larger numbers:

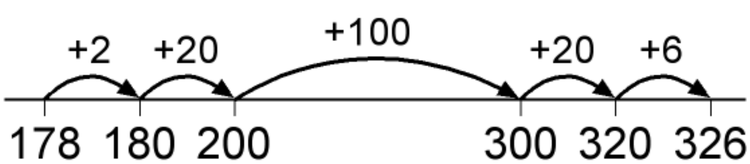
45-32=

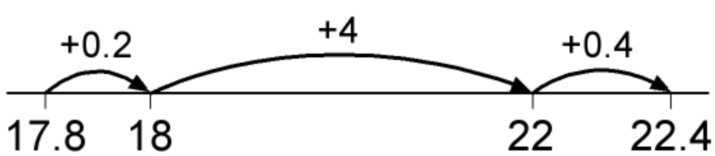


Stage 5

Some children may find it easier to do certain subtraction calculations by counting up (usually to find the 'difference' or to find change when dealing with money)







Stage 6

Expanded partitioning layout

Crossing no boundaries

77-24=

70	7
20	4
50	3

Crossing boundaries, requiring decomposition

74-27=

60 ~~70~~	4
20	7
40	7

Stage 7

Compact decomposition method

73-45=

$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}3 \\ 4\ \ 5 \\ \hline 2\ \ 8 \end{array}$$

Multiplication

Following practical and visual activities

Stage 1

Bar model – to understand and reason what multiplication means.

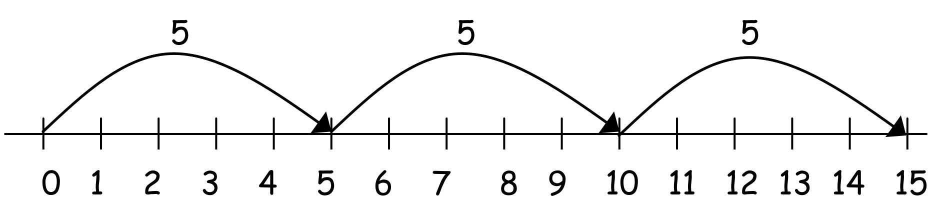
20				
4	4	4	4	4

4 x 5 = 20   20 = 4 x 5

Stage 2

Repeat addition on a number line

3 x 5 = 5 + 5 + 5



Stage 3

Partitioning using known facts

14x5=

$$\begin{array}{r} 10\times5=50 \\ 4\times5=20 \\ \hline 70 \end{array}$$

23x4=

$$\begin{array}{r} 10\times4=40 \\ 10\times4=40 \\ 3\times4=12 \\ \hline 92 \end{array}$$

Stage 4

The grid method

TUxU 45x3=

x	40	5
3	120	15

> 135

TUxTU 23x34=

x	20	3
30	600	90
4	80	12

```
>690
> 92
 782
```

Stage 5

Expanded layout

56x27=

```
    56
 X  27
 -----
  1000    (50x20)
   120    (6x20)
   350    (50x7)
    42    (6x7)
 -----
  1512
```

Stage 6

Compact method

56x27=

```
      5  6
 X  42  7
 ---------
   13  9  2
 1 11  2  0
 ---------
 1  5  1  2
```

Division

Following practical and visual activities:

Stage 1

Bar model – to understand and reason what division means.

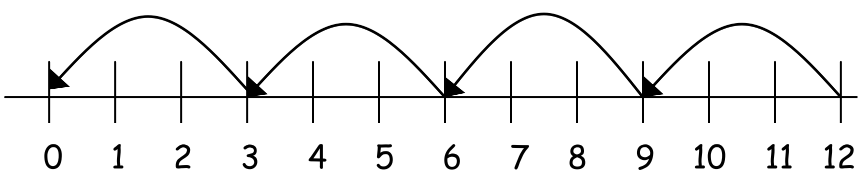
20				
4	4	4	4	4

20 ÷ 5 = 4

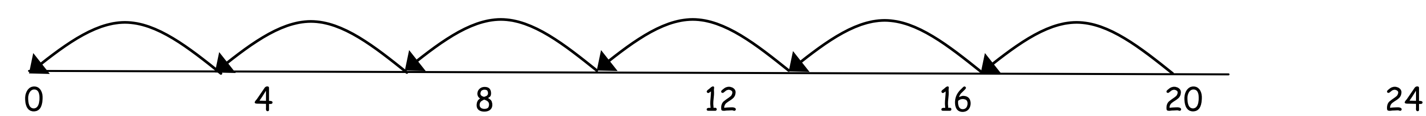
Stage 2

Repeat subtraction on a number line

12 ÷ 3 = 4



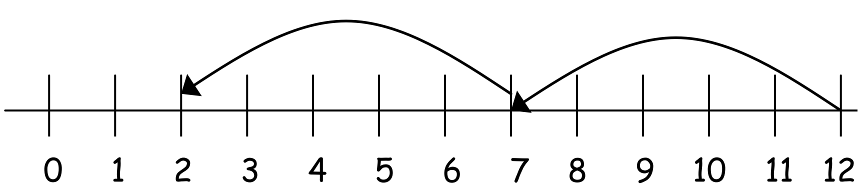
24 ÷ 4 = 6



Stage 3

Use a number line to repeatedly subtract and find remainders

12÷5=2r2



Stage 4

Division using partitioning (children must be able to recognise multiples of a number and partition numbers in different ways)

84÷7=

$$
\begin{aligned}
84\div7 &= (70+14)\div7\\
70\div7 &= 10\\
14\div7 &= \underline{\ \ 2\ \ }\\
&\quad 12
\end{aligned}
$$

Stage 5

Chunking

96 ÷ 6

$$
\begin{array}{r|l}
 & \ \ 1\,6 \\ \hline
6 & 96 \\
 & -\underline{60}\qquad 10 \times 6 \\
 & \ \ 36 \\
 & -\underline{36}\qquad 6 \times 6 \\
 & \ \ \ 0
\end{array}
$$

Answer:16

196 ÷ 6

$$
\begin{array}{r|l}
 & \ 32 \text{ r } 4 \\ \hline
6 & 196 \\
 & -\underline{180}\qquad 30 \times 6 \\
 & \ \ \ 16 \\
 & -\underline{\ \ 12}\qquad 2 \times 6 \\
 & \ \ \ \ \ 4
\end{array}
$$

Answer 32r4

Stage 6
Bus stop

642÷3=

```
   2  1  4
3 | 6  4 12
```

643 ÷3=

```
   2  1  4 r 1
3 | 6  4 13
```

Stage 7
Long Division